4.7 L'Hospital's Rule

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alt L'Hôpital's or L'Hopital's

indeterminate forms

$\frac{0}{0}$, $\frac{\infty}{\infty}$

if $f(x)$ and $g(x)$ are continuous & differentiable functions,

and if $\lim\limits_{x \to \infty} \frac{f(x)}{g(x)}$ exists, but the limit initially has a form of $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$x \to -\infty$, $x \to a$

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{f'(x)}{g'(x)}$$

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WARNING

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→ this does NOT say to use the quotient rule.

→ MUST be indeterminate

$\frac{0}{0}$ or $\frac{\infty}{\infty}$

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$$2(x+4) = 2x + 2\cdot 4$$

$$2(4x) \stackrel{?}{=} 2\cdot 4\cdot 2x$$

$$\lim_{x\to\infty} \frac{x^2+x-7}{x^2+2x+180}$$

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$$\lim_{x\to\infty} \frac{2x^2+x-7}{x^2+2x+180}$$

$$=\lim_{x\to\infty} \frac{2x+1}{2x+2} = \lim_{x\to\infty} \frac{2}{2} = \underline{1}$$

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Sections on L'Hospital's Rule
include discussions of
5 more indeterminate form

Really interesting use

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x^2 vs x

$$\lim_{x\to\infty} \frac{x^2}{x} = \lim_{x\to\infty} x = \infty$$

$$\lim_{x\to\infty} \frac{x+7}{x-2} = 1$$

ln x vs x

$$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{\left(\frac{1}{x}\right)}{1} = 0$$

$$\lim_{x\to\infty} \frac{x}{\ln x} \stackrel{LH}{=} \lim_{x\to\infty} \frac{1}{\left(\frac{1}{x}\right)} = \lim_{x\to\infty} x = \infty$$

$Y_1 = (\ln(x))/x$

quit

$Y_1(1000$

$\ln(1000)$

1000

Riemann Zeta Fn

Music of the Primes

Marcus du Sautoy

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Compare e^x to x

$$\lim_{x\to\infty} \frac{e^x}{x} \overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{1} = \lim_{x\to\infty} e^x = \infty$$

Compare e^x to $x^2, x^3, \ldots$

Compare e^x to x^n

$$\lim_{x\to\infty} \frac{e^x}{x^n} \overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{nx^{n-1}} \overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{n(n-1)x^{n-2}}$$

$$\overset{LH}{=} \cdots \overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{n(n-1)(n-2)\cdots 2x}$$

$$\overset{LH}{=} \lim_{x\to\infty} \frac{e^x}{n!}$$

e^x grows MUCH faster than ANY polynomial of ANY degree

Compare $\ln x$ to $\sqrt{x}$

4.5/9 show that the linear approximation to T at the point $x=0$ is day 2^6

. . . .

[given $T(x) = 60D(60+x)^{-1}$]

Recall: the eqn of the tangent line is $y - T(0) = T'(0)(x-0)$

$+T(0) \quad +T(0)$

THE LINEAR APPROXIMATION IS THE TANGENT LINE.

$y = T(0) + T'(0)(x)$

$T(0) = 60D(60+0)^{-1} = \frac{60D}{60} = D$ ✓

$T'(x) = \frac{d}{dx}\left(60D(60+x)^{-1}\right)$

$= -60D(60+x)^{-2}$

$T'(0) = -60D(60)^{-2} = \frac{-60D}{60^2} = -\frac{D}{60}$ ✓

$y = D - \frac{D}{60}x = D\left(1 - \frac{x}{60}\right)$

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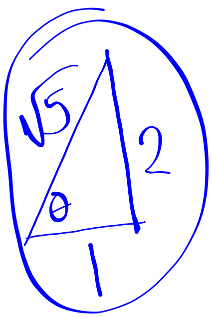
4.1/69 $\quad f(x) = 2\sin x + \cos x$ $\quad$ day 2^6

$[-2\pi, 2\pi]$

CP EV

$f'(x) = 2\cos x - \sin x$

$f'(x) = 0 \Rightarrow$ $\quad$ [[$f'(x)$ is never undefined]]



$2\cos x - \sin x = 0$

$\dfrac{2\cos x}{\cos x} = \dfrac{\sin x}{\cos x}$

$2 = \tan x$, so $x = \arctan(2)$ ✗ domain

2ND deriv test $\quad f''(x) = \frac{d}{dx}(2\cos x - \sin x)$

$= -2\sin x - \cos x$

$f''(\arctan(2)) = -2\left(\frac{2}{\sqrt{5}}\right) - \left(\frac{1}{\sqrt{5}}\right) =$

$-\frac{5}{\sqrt{5}} = -\sqrt{5}$

$f''(\arctan(2))$ NEGATIVE so $\arctan(2)$ is a rel MAX

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$f(x) = 1 - |x|$; $[-1, 1]$

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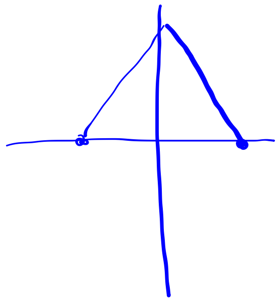

$f(-1) = 1 - |(-1)| = 1 - 1 = 0$

$f(1) = 1 - |1| = 1 - 1 = 0$

$f(x)$ is continuous on $[-1, 1]$

$f(x)$ is NOT differentiable on $[-1, 1]$

so I can't use MVT or Rolle's Theorem.

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